



## Enriched Galerkin Discretization for Modelling Flow in Fractured Porous Media using Mixed-Dimensional Approach

Kadeethum, Teeratorn; Nick, Hamid; Richardson, C. N.; Ballarin, F.; Lee, S.

*Publication date:*  
2019

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Kadeethum, T., Nick, H., Richardson, C. N., Ballarin, F., & Lee, S. (2019). *Enriched Galerkin Discretization for Modelling Flow in Fractured Porous Media using Mixed-Dimensional Approach*. Abstract from FEniCS'19, Washington DC, District of Columbia, United States.

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# FENICS `19: FENICS `19

PROGRAM AUTHORS KEYWORDS

## PROGRAM FOR THURSDAY, JUNE 13TH

Days: [previous day](#) [next day](#) [all days](#)

View: [session overview](#) [talk overview](#)

**09:00-09:30** Coffee/Tea

LOCATION: [Greenwalt Common Room](#)

**09:30-11:00** Session 6: Numerical methods I

CHAIR: [Johan Jansson](#)

LOCATION: [Greenwalt Auditorium](#)

09:30 [Ingeborg Gjerde](#), [Kundan Kumar](#) and [Jan Martin Nordbotten](#)

### **Analysis and Discretization of Coupled 1D-3D Flow Models**

**ABSTRACT.** Coupled 1D-3D flow models are used for a variety of applications, such as modelling fluid flow through vascularized tissue, modelling the flow of water and nutrients through soil embedded with a root system, or modelling the interaction between a well and reservoir. Veins, arteries, roots and wells all have in common that their radius is negligible compared to their length and the size of the domain as a whole. For this reason, we idealize them as being 1D geometries. The 1D structures are then endowed with a 1D flow equation, and coupled to the 3D flow equation by the use of a line source.

The main challenge associated with coupled 1D-3D flow models is that the line source makes the solution singular. This complicates both the analysis and approximation of the solution. In this talk, we show that the solution admits a splitting into two parts: (i) a term that explicitly captures the singularity and (ii) some smooth remainder. Via this splitting, we can then subtract the singularity. This yields a reformulated model in which all variables are smooth. The solution can then be approximated using any standard numerical method. We conclude by showing numerical experiments relevant to biomedical applications.

09:50 [T. Kadeethum](#), [H. M. Nick](#), [C. N. Richardson](#), [F. Ballarin](#) and [S. Lee](#)

### **Enriched Galerkin Discretization for Modelling Flow in Fractured Porous Media using Mixed-Dimensional Approach**

PRESENTER: [T. Kadeethum](#)

**ABSTRACT.** Fluid flow and solute transport in fractured porous media is the backbone of many applications including groundwater flow, underground energy harvesting, earthquake prediction, and biomedical engineering. The traditional continuous Galerkin (CG) method is not suitable for the transport equation due to lack of mass conservation. The discontinuous Galerkin (DG) method mitigates this problem; however, its computational cost is considerably more than the CG method. In this study, a robust and efficient discretization method based on the incomplete interior penalty enriched Galerkin (EG) method is proposed. This method requires fewer degrees of freedom than those of the DG method, while it achieves the same accuracy. The flow and transport models of rock matrix and fractures domains are investigated in the mixed-dimensional setting. The results of combinations of function spaces, for example, (i)  $CG \times CG$ , (ii)  $CG \times EG$ , and (iii)  $CG \times DG$  spaces are compared. The results illustrate the superiority of the EG and DG methods in solving the flow and transport equations in fractured porous media. Furthermore, the computational burden of the EG method is two times cheaper than that of the DG method.

- 10:10 [Hannah Morgan](#) and [Ridgway Scott](#)  
**Improved modeling of certain non-Newtonian fluids**  
 PRESENTER: [Hannah Morgan](#)

**ABSTRACT.** Non-Newtonian fluids are found in all aspects of life, from bodily fluids to engine oil. Thus advances in modeling and simulation of non-Newtonian fluids can have broad impact. Models of non-Newtonian fluids have been studied extensively for many years, but only recently have there been mathematical advances that allow models for them to be understood more completely. This understanding now allows development of numerical solution methods with a new level of reliability. Here we consider some Oldroyd models and their relation to the grade-two model. In this talk, we present a computational study of a solution method using tools from the FEniCS Project to facilitate code generation and to support correctness of the implementation.

- 10:30 [Olalekan Babaniyi](#), [Omar Ghattas](#), [Noemi Petra](#) and [Umberto Villa](#)  
**hiPPYlib: An Extensible Software Framework for Large-Scale Inverse Problems**  
 PRESENTER: [Olalekan Babaniyi](#)

**ABSTRACT.** We present an Inverse Problem Python library (hiPPYlib) for solving large-scale deterministic and Bayesian inverse problems